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A Comparative Assessment of Local and Nonlocal Damage Models for Prediction of Fracture Behavior during Mixed-Mode Loading

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Abstract

For components made of ductile materials, finite element approaches which incorporate material damage constitutive models are able to predict the failure behaviour and the corresponding process with high accuracy. However, these local approaches suffer from the problem of mesh-dependency of the results. Problem arises when one needs to simulate large stress gradients and model miniaturized specimens with a pre-determined mesh-size. Nonlocal regularization of the material state variables can alleviate this problem and this has been investigated by various researchers over the years. Recently, the authors have developed a nonlocal form of the Rousselier's damage model and have used it to demonstrate that the results of the model are independent of mesh size. In this work, the issue of simulation of mixed-mode loading with the damage models will be discussed. The crack growth and the fracture resistance behavior of a standard compact tension specimen, loaded in mode-I and mixed mode, have been simulated with the use of local as well as the nonlocal damage models. In case of mixed mode loading, the calculation with the local model predicts a crack growth direction nearly parallel to the crack plane, which is not physical and it contradicts with experimental observation. This issue has been resolved with the help of the nonlocal formulation.

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Keywords: Nonlocal damage model; Rousselier's model; ductile fracture; finite element simulation

1. Introduction

For components fabricated from ductile materials (e.g., reactor grade steels), finite element approaches which incorporate material damage constitutive models are able to predict the failure behavior and the corresponding process with high accuracy [1]. In the last two decades, damage models like Rousselier's or Gurson's model (and its modified versions) have become powerful tools in the safety analysis of critical plant components. These damage models predict the material behaviour on the basis of the micromechanical

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processes (i.e., void nucleation, growth and coalescence) leading to ductile fracture [2]. Due to the local nature of the formulation, the damage tends to localize in one element layer. As a result of this localization, the finite element results become mesh-dependent. Problems also arise when it is required to simulate high stress gradients or small amounts of crack growth using these damage models.

In order to overcome this problem, nonlocal and gradient based formulations have been developed by various researchers all over the world [3]. Recently, the authors have developed a nonlocal form of the Rousselier's damage model and have used it to demonstrate that the results of the model are independent of mesh size [4]. In this work, the issue of simulation of mixed-mode loading condition with the help of continuum damage models will be discussed.

In engineering applications, the pre-cracked components are often loaded in mixed mode. A pure mode-I load leads to a crack growth parallel to the crack plane. The superposition of a mode-II load on mode-I load leads to deflection of the crack from the crack plane. For the simulation of a mixed-mode loading condition, the specimen geometry proposed by Richard and Benitz [5] has been used in this work.

The crack growth and the fracture resistance behavior of this specimen have been simulated with the use of local as well as the nonlocal damage models. The calculation with the local model predicts a crack growth direction nearly parallel to the crack plane, which is not physical and it contradicts with experimental observation. This issue has been resolved with the help of the nonlocal formulation. The details of the results of both the local and nonlocal models and their comparative assessment will be presented in this paper.

2. Nonlocal Rousselier's damage model

Recently, the authors have developed a nonlocal formulation of Rousselier's model using nonlocal damage d as a nodal degree of freedom in the FE mesh. The increment of the nonlocal damage variable \dot{d} in a material point \bar{x} is mathematically defined as a weighted average of the increment of the local void volume fraction \dot{f} in a domain Ω [Fig. 1], i.e.,

$$\dot{d}(\bar{x}) = \frac{1}{\Psi(\bar{x})} \int_{\Omega} \Psi(\bar{y}; \bar{x}) \dot{f}(\bar{y}) d\Omega(\bar{y}) \quad (1)$$

where \bar{y} is the position vector of the infinitesimally small volume $d\Omega$ and $\Psi(\bar{y}; \bar{x})$ is the Gaussian weight function given by

$$\Psi(\bar{y}; \bar{x}) = \frac{1}{8\pi^{3/2}l^3} \exp\left(-\frac{|\bar{x} - \bar{y}|^2}{4l^2}\right) \quad (2)$$

The length parameter l in Eq. (2) determines the size of the volume, which effectively contributes to the nonlocal quantity and is related to the scale of the microstructure. The above integral nonlocal kernel holds the property that the local continuum is retrieved if $l \rightarrow 0$. By expanding $f(\bar{y})$ in Taylor's series and substituting in Eq. (1) and doing some algebra, one can obtain the damage diffusion equation as

$$\dot{d} - \dot{f} - C_{length} \nabla^2 \dot{d} = 0 \quad (3)$$

where C_{length} is the characteristic length parameter of the material. The yield function of the Rousselier's model [2] is modified by substituting the nonlocal damage d in place of the local ductile void volume fraction f as

$$\phi = \frac{q}{1-d} + D\sigma_k d \exp\left(\frac{-p}{(1-d)\sigma_k}\right) - R(\epsilon_{eq}) = 0 \quad (4)$$

where D and σ_k are the parameters of the Rousselier's model and are constants for a material. With loading, the void volume fraction evolves from the initial void volume fraction f_0 (volume fraction of eligible second phase particles responsible for nucleation of voids upon plastic deformation) in the material.

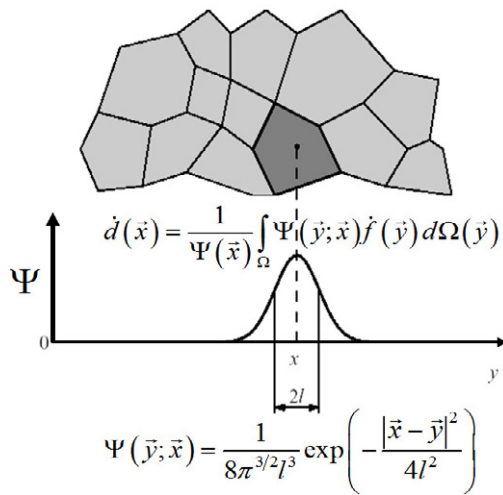


Fig. 1. Regularization of the internal variable 'f' through a weighted integral in a volume Ω

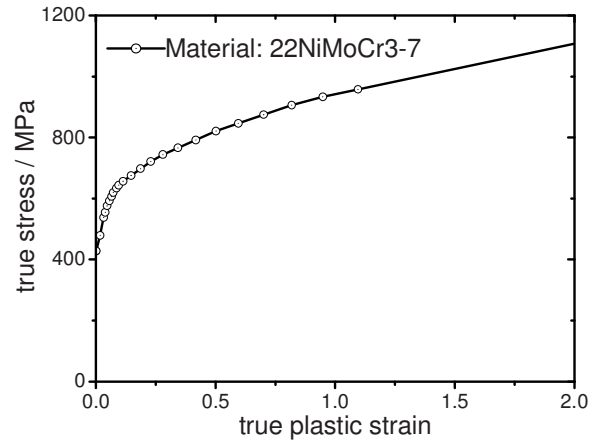


Fig. 2. True stress vs. true plastic strain curve of the material DIN 22NiMoCr3-7.

At a critical void volume fraction f_c , the voids coalesce with each other and at the final void volume fraction f_f , the material points loses its stress carrying capability. Hence, the above three void parameters (i.e., f_0, f_c and f_f) are also the material properties of the damage model. For solving the boundary value problem of the nonlocal damage continuum, one needs to solve the partial differential equation (3) along with the mechanical equilibrium equation

$$\nabla \cdot \sigma + f_b = 0 \quad (5)$$

and the associated boundary conditions, where σ is the Cauchy stress tensor and f_b is the body force per unit volume. For the nonlocal damage degree of freedom, the additional (i.e., Neumann) boundary condition is used and is expressed as

$$\nabla \dot{d} \cdot n|_{\Gamma_d} = 0 \quad (6)$$

Where $n|_{\Gamma_f}$ is the normal to the boundary Γ_f . By discretizing the weak forms of the governing differential equations (3) and (5), we obtain the FE equations in matrix form as

$$\begin{bmatrix} K_{uu} + K_{NL} & K_{ud} \\ K_{du} & K_{dd} \end{bmatrix} \begin{Bmatrix} \Delta \hat{u} \\ \Delta \hat{d} \end{Bmatrix} = \begin{Bmatrix} f_m^{ext} - f_m^{int} \\ -f_d^{int} \end{Bmatrix} \quad (7)$$

It may be noted that the stiffness terms K_{ud} , K_{du} and K_{dd} in the element stiffness matrix are contributions of the nonlocal formulation. The matrix K_{uu} represents the conventional stiffness of the finite element and it represents the mechanical stiffness, which corresponds to the relationship of the nodal mechanical forces and nodal displacement vectors. Similarly, K_{ud} represents the matrix, which produces nodal damage vector

corresponding to applied mechanical force vector. The matrix K_{du} produces nodal displacement vector corresponding to an applied damage force vector and K_{dd} produces nodal damage vector corresponding to applied damage force vector respectively. f_m^{int} and f_m^{ext} are the internal and external mechanical force vectors respectively, whereas f_d^{int} is the internal damage force vector.

3. Results and discussion

The programming and simulation efforts for a non local damage model are considerably higher than those required for a corresponding local model. However, there are obvious advantages of the nonlocal formulation and these will be discussed in this section in the context of simulation of different types of loading conditions on the fracture mechanics specimens. In an earlier work, the standard 1T compact tension (CT) specimen was simulated to obtain the load-displacement and the fracture resistance behaviour. The effect of the finite element mesh size (in the crack tip region) on the load-displacement behaviour of the CT specimen was studied using both local and nonlocal damage models. It was observed that the results of the nonlocal models are mesh-independent in contrast to those of local model. The results of the nonlocal simulation also compare very well with those of experiment. For all these analyses, the material properties of the German pressure vessel steel DIN 22NiMoCr3-7 were used. This material is homogeneous from the point of chemical composition and possesses good ductility.

Table. 1. Material properties of DIN 22NiMoCr3-7 pressure vessel steel.

Model parameters	Rousselier's constants		Void parameters			Char. length parameter	E (Young's mod. in GPa)	ν (Poisson's ratio)
	D	σ_k (MPa)	Initial void volume fraction f_0	Void volume fraction at coalescence f_c	Final void volume fraction f_f	C_{length} (mm ²)		
Value	2	445	0.0003	0.05	0.3	0.05	210	0.3

The true stress-strain curve of the material used in FE analysis is shown in Fig. 2. Other material properties including the Rousselier's constants, characteristic length parameter and the void volume fractions (initial, final and at coalescence) are shown in Table-1. In engineering applications, the pre-cracked components are often loaded in mixed mode. A pure mode I load leads to a crack growth parallel to the crack plane. The superposition of a mode II load on mode I load leads to deflection of the crack from the crack plane.

For the simulation of a mixed mode load, the specimen geometry proposed by Richard and Benitz [5], has been used in this work, Fig. 1(a). The specimen type will be called as CTS specimen in the following discussion. The behaviour of this specimen type has been simulated with the use of local as well as the nonlocal damage models. The finite element mesh used in the simulation is shown in Fig. 3(b).

For the CTS specimen with a pure mode I load, both the local and the nonlocal models predict a crack growth path parallel to the crack plane, Figs. 4(a-b). If the specimen is loaded asymmetrically at an angle of 45° to the horizontal direction, the crack is stressed with a mixed mode loading condition. The calculation with the local model predicts a crack growth direction nearly parallel to the crack plane, Fig. 4(c), which is not physical and contradict with experimental observation. The nonlocal model however predicts a crack growth direction which deflects clearly from the symmetry plane (which would typically be expected in an experiment under plane strain condition), Fig. 4(d).

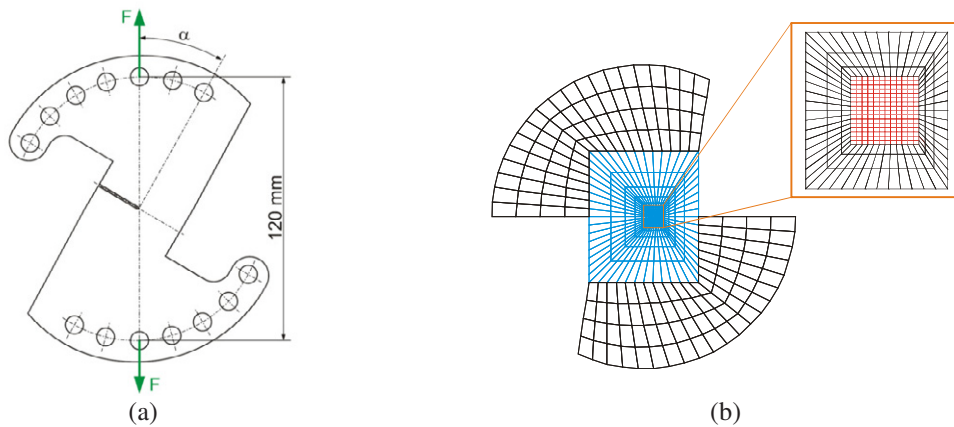


Fig. 3. Richards's CTS specimen. (a) Geometry and loading configuration (α controls the loading for mixed mode. α is zero for pure mode I and 90° for pure mode II loading condition); (b) Finite element mesh and its enlarged view near the crack-tip.

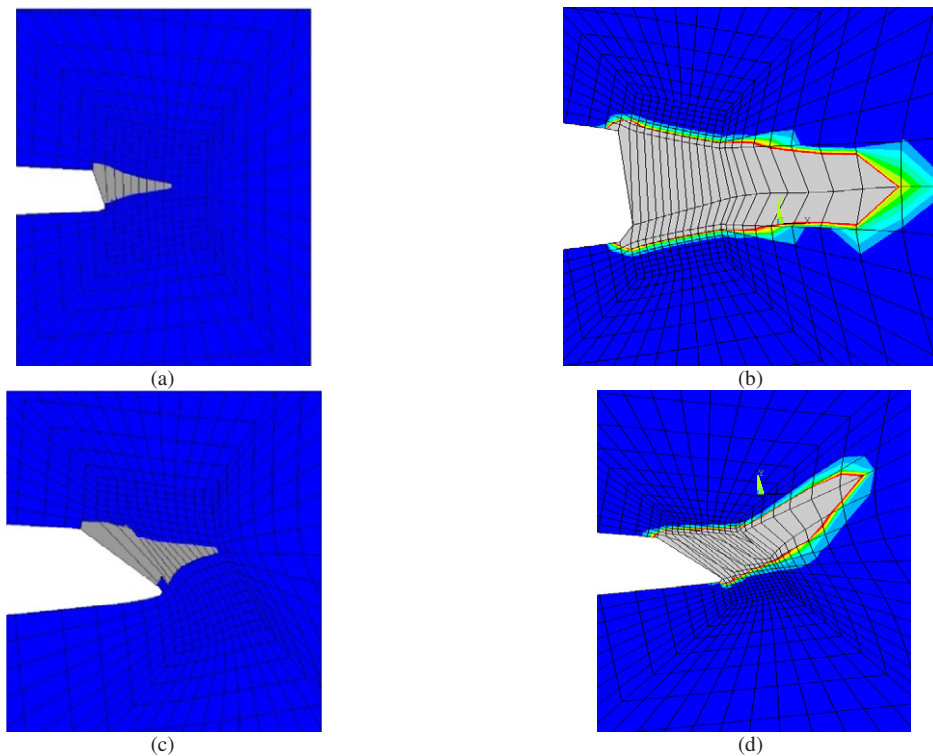


Fig. 4. Damage zone as predicted by the local and nonlocal Rousselier's damage model for different modes of loading of Richards's CTS specimen. (a) mode-I loading, local model; (b) mode-I loading, nonlocal model; (c) mixed-mode loading, local model; (d) mixed-mode loading, nonlocal model.

The Influence of mixed-mode loading on the predicted load elongation behaviour of the CTS specimen is shown in Fig. 5 for both local and nonlocal damage models. The results of elastic-plastic analysis (without damage) for mode-I and mixed-mode loading cases are also shown in Fig. 5 for the purpose of comparison.

It can be noted that there is load-drop in the response predicted by both the local and non-local model for mode-I loading. There is no load-drop in the response predicted by the elastic-plastic analysis for both mode-I and mixed-mode loading situations and this is as per expectation, because of non-consideration of material degradation in the constitutive equations. However, for mixed-mode loading, the local damage model does not

predict much load drop. This is because the crack path is not correctly predicted and hence, there is no significant crack growth.

However, the nonlocal model is able to predict the load-drop successfully and it is because of the ability of the model to predict the accurate crack path depending upon the mixed-mode state of stress at the crack tip. Hence, it is demonstrated that the local damage models are quite inadequate in many arbitrary loading situations where the crack path and loading state of stress are not known in advance and hence, one cannot make suitable mesh design to capture the crack path (this is in addition to its inadequacy with respect to handling of arbitrary fine mesh size). The nonlocal model inherently captures this aspect and hence is of considerable appeal for a reliable safety analysis.

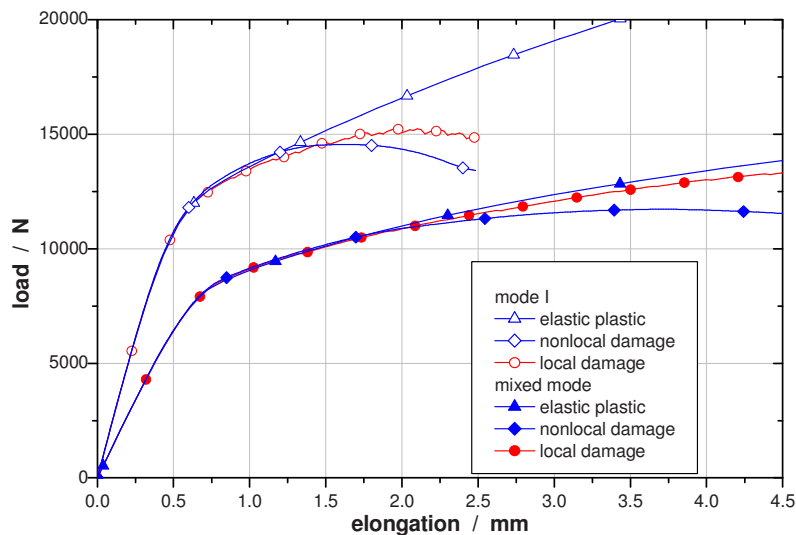


Fig. 5. Influence of mixed model loading on the predicted load elongation behaviour of a CTS specimen.

4. Conclusions

Though local damage models for ductile crack growth simulation have been very successful in predicting the load-deformation and fracture resistance behaviour of different types of specimens and components, it is not able to simulate the condition of large stress gradients. This is due to the mesh-dependence nature of the solutions. It is demonstrated that the local damage models are quite inadequate in many arbitrary loading situations including the mixed-mode loading of the crack-tip, where the crack path and loading state of stress are not known in advance. The nonlocal model inherently captures this aspect and hence is of considerable appeal for a reliable safety analysis.

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